ISEN 629 **Engineering Optimization**
Fall 2007

Homework # 1

Due Date: September 20, Thursday.

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1. Show that the function $\bar{f}(x)$ in the proof of Theorem 1.1.2 from the text is l_∞-Lipschitz continuous with the constant L and its global optimal value is $-\epsilon$.

2. Show that the following functions belong to $\mathcal{F}^1(\mathbb{R})$:

   (a) $f(x) = e^x$;

   (b) $f(x) = |x|^p, \ p > 1$;

   (c) $f(x) = \frac{x^2}{1+|x|}$;

   (d) $f(x) = |x| - \ln(1 + |x|)$.

3. Show that for given $a_i \in \mathbb{R}^n, b_i \in \mathbb{R}$ $(i = 1, \ldots, m)$ the following functions belong to $\mathcal{F}^1(\mathbb{R}^n)$:

   (a) $f(x) = \sum\limits_{i=1}^{m} e^{a_i^T x + b_i}$;

   (b) $f(x) = \sum\limits_{i=1}^{m} |a_i^T x - b_i|^p, \ p > 1$.

4. Prove theorem 2.1.6.

5. (defining the class of strongly convex functions) Consider a hypothetical class of functions $\mathcal{F}$ satisfying the following assumptions:

   (a) For any $f \in \mathcal{F}$ there exists some constant $\mu > 0$ such that for any $\bar{x}$ with $f'(\bar{x}) = 0$ and any $x \in \mathbb{R}^n$ we have

   $$f(x) \geq f(\bar{x}) + \frac{1}{2}\mu\|x - \bar{x}\|^2.$$

   (b) If $f_1, f_2 \in \mathcal{F}$ and $\alpha, \beta \geq 0$, then $\alpha f_1 + \beta f_2 \in \mathcal{F}$.

   (c) Any strictly convex quadratic function $f(x) = x^T Ax + b^T x$, where Q is positive definite, belongs to $\mathcal{F}$.

   Show that any function from class $\mathcal{F}$ has the following property: there exists a constant $\mu > 0$ such that for any $x, y \in \mathbb{R}^n$ we have

   $$f(y) \geq f(x) + f'(x)^T(y - x) + \frac{1}{2}\mu\|y - x\|^2.$$