ISEN 629 **Engineering Optimization**
Fall 2007

Homework # 2

Due Date: October 18, Thursday.

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1. Prove Theorem 2.1.9.

2. Prove Theorem 2.1.11.

3. Use a one-dimensional example to illustrate Theorem 3.1.1 geometrically.

4. Given two convex functions f_1 and f_2 on $\mathbb{R}^n$, define their convolution function as

$$f(x) = \inf_y \{f_1(x - y) + f_2(y)\}.$$

   Show that $f(x)$ is convex.

5. A function $f : X \to [-\infty, +\infty]$, where $S \subseteq \mathbb{R}^n$ is called lower semi-continuous at a point $x \in S$ if

$$f(x) \leq \lim_{i \to \infty} f(x_i)$$

   for every sequence $\{x_i : i \geq 0\} \subseteq S$ such that $x_i \to x, i \to \infty$, and the limit of $\{f(x_i) : i \geq 0\}$ exists in $[-\infty, +\infty]$.

   For an arbitrary function $f : \mathbb{R}^n \to [-\infty, +\infty]$, prove that the following conditions are equivalent:

   (a) f is lower semi-continuous throughout $\mathbb{R}^n$;

   (b) $\mathcal{L}_f(\alpha) = \{x : f(x) \leq \alpha\}$ is closed for every $\alpha \in \mathbb{R}$;

   (c) $\text{epi}(f)$ is a closed set in $\mathbb{R}^{n+1}$.