ISEN 629 **Engineering Optimization**
Fall 2007

Homework # 3

Due Date: November 1, Thursday.

1. Consider the convex function f defined on $\mathbb{R}^n$ by
$$f(x) = \begin{cases} +\infty, & x < -1 \\ 2, & x = -1 \\ x^2, & -1 < x \le 0 \\ x, & 0 \le x \le 1 \\ +\infty, & 1 < x. \end{cases}$$
Compute $\partial f(x)$.

2. Prove Lemma 3.2.1.

3. Consider the problem $\min\{f(x) : x \in Q\}$. Let $\{x_i : i \geq 0\}$ be a sequence in Q. Define by
$$S_k = \{x \in Q : g(x_i)^T(x_i - x) \geq 0, i = 0, \ldots, k\}.$$
the localization set of our problem generated by the sequence $\{x_i : i \geq 0\}$. Let x^* be a solution to our problem. Then for all $k \geq 0$ we have $x^* \in S_k$. Denote by
$$v_i = v_f(x^*, x_i), \quad v_k^* = \min_{0 \le i \le k} v_i.$$
Prove that $v_k^* = \max\{r : g(x_i)^T(x_i - x) \geq 0, i = 0, \ldots, k, \forall x \in B_r(x^*)\}$.

4. Prove Theorem 3.2.2.

5. Denote by
$$\Delta_k = \frac{R^2 + \sum\limits_{i=0}^{k} h_i^2}{2\sum\limits_{i=0}^{k} h_i},$$
where R is some constant.
   (a) $\Delta_k \to 0$ if $\sum\limits_{i=0}^{\infty} h_i = \infty$.
   (b) Δ_k is a convex function of $\{h_k\}_{k=0}^N$.
   (c) Minimizing Δ_k as a function of $\{h_k\}_{k=0}^N$ yields the following solution:
$$h_i = \frac{R}{\sqrt{N+1}}, \; i = 0, \ldots, N.$$

6. Prove that $v_{\bar{f}}(x^*; x) \geq 0$ for all $x \in \mathbb{R}^n$ as described on slide 18 of Lecture 15.