

ISEN 629 **Engineering Optimization**
Fall 2007

Homework # 5

Due Date: December 6, Thursday.

1. Write down the step of a path-following scheme for the linear optimization problem

$$\begin{aligned} & \min c^T x, \\ \text{s.t. } & Ax = b, \\ & x \geq 0 \quad (\Leftrightarrow x \in \mathbb{R}_+^n), \end{aligned}$$

where A is an $m \times n$ -matrix, $m < n$. Use the standard logarithmic barrier for \mathbb{R}_+^n :

$$F(x) = - \sum_{i=1}^n \ln x_i, \quad \nu = n.$$

What is the complexity of one Newton step? What is the total arithmetical complexity of the algorithm? (Hint: follow the analysis of the path-following scheme for SDP in the text).

2. Show that the function

$$F(x, t) = - \ln x - \ln(\ln x + t)$$

is a 2-self-concordant barrier for the set

$$Q = \{(x, t) \in \mathbb{R}^2 \mid x > 0, t \geq -\ln x\}.$$

3. Write down the barrier model of the problem of finding a minimum volume ellipsoid inscribed in a convex polytope (with free center). Explain your model. (Hint: clarify section 4.3.4.3, in particular the use of Lemma 4.3.8)